A Closer Look at the S-box: Deeper Analysis of Round-Reduced ASCON-HASH

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Abstract. ASCON, a lightweight permutation-based primitive, has been selected as NIST's lightweight cryptography standard. ASCON-HASH is one of the hash functions provided by the cipher suite ASCON. At ToSC 2021, the collision attack on 2-round ASCON-HASH with time complexity 2^{103} was proposed. Due to its small rate, it is always required to utilize at least 2 message blocks to mount a collision attack because each message block is only of size 64 bits. This significantly increases the difficulty of the analysis because one almost needs to analyze equivalently at least 2ℓ rounds of ASCON in order to break ℓ rounds. In this paper, we make some critical observations on the round function of ASCON, especially a 2-round property. It is found that such properties can be exploited to reduce the time complexity of the 2-round collision attack to $2^{62.6}$. Although the number of attacked rounds is not improved, we believe our techniques shed more insight into the properties of the ASCON permutation and we expect they can be useful for the future research.

Keywords: ASCON · ASCON-HASH · Collision Attack · Algebraic Technique

1 Introduction

Lightweight cryptography algorithms are a class of ciphers designed for resource-constrained environments. They typically have low requirements in terms of computing power, storage space, and power consumption, and are suitable for resource-constrained application scenarios such as embedded systems, IoT devices, and sensors.

In 2013, NIST started the lightweight cryptography project. Later in 2016, NIST provided an overview of the project and decided to seek for some new algorithms as a lightweight cryptography standard. In 2019, NIST received 57 submissions and 56 of them became the first round candidates after the initial

review. After the project proceeded into Round 2 [4], NIST selected 32 submissions as Round 2 candidates, including ASCON. After that, ASCON was selected to be one of the ten finalists of the lightweight cryptography standardization process. On February 7, 2023, NIST announced the selection of the ASCON family for the lightweight cryptography standardization.

ASCON [10] is a lightweight permutation-based primitive. It aims to provide efficient encryption and authentication functions while maintaining sufficiently high security.

Advantages. The main advantages of ASCON can be summarized as below:

- **Lightweight**: The design of ASCON is simple and suitable for hardware and software implementation. It is particularly suitable for resource constrained environments, such as IoT devices, embedded systems, and low-power devices.
- **High security**: ASCON provides high security and resists many different types of known attacks.
- **Adjustable**: ASCON supports different security levels and performance requirements. For example, ASCON-128 and ASCON-128a provide a 128-bit security level, suitable for high security requirements; ASCON-80pq provides an 128-bit security level, suitable for low-power and low-cost scenarios.
- **Authentication encryption**: ASCON can achieve both data encryption and integrity protection. It supports associated data and allows for verification of additional information during the encryption process, such as the identities of message senders and receivers.

History. ASCON was first published as a candidate in Round 1 [6] of the CAESAR competition [1]. This original design (version v1) specified the permutation as well as the mode for authenticated encryption with two recommended family members: The primary recommendation Ascon-128 as well as a variant Ascon-96 with 96-bit key. For the subsequent version V1.1 [7] and V1.2 [8], minor functional tweaks were applied, including a reordering of the round constants and the modification of the secondary recommendation to the current Ascon-128a. Then, V1.2 [8] and the status update file [9] were submitted to the NIST Lightweight Cryptography project. The submission to NIST includes not only the authenticated cipher family, but also introduces modes of operation for hashing: ASCON-HASH and ASCON-XOF, as well as a third parameterization for authenticated encryption: Ascon-80pq. For ASCON-HASH and ASCON-XOF, they support 256-bit and arbitrary-length hash values, respectively.

On the collision resistance of ASCON-HASH. Due to the used sponge structure, the generic time complexity to find a collision of ASCON-HASH is 2^{128} and the memory complexity is negligible with Floyd's cycle finding algorithm [11]. Due to its small rate, it is quite challenging to find collisions for a large number of rounds. In [21], the first 2-round collision attack on ASCON-HASH was presented with time complexity 2^{125}. However, it is shown that such an attack is invalid because the used 2-round differential characteristic is invalid according to [13].

Later, at ToSC 2021 [12], a new and valid 2-round differential characteristic with an optimal differential probability was found. Based on the same attack strategy as in [21], they gave a 2-round collision with time complexity of 2^{103} in [12]. Very recently, Qin et al. presented collision attacks on 3 and 4 rounds of ASCON-HASH by turning preimages for ASCON-XOF into collisions for ASCON-HASH [19]. However, it can be found that both the time complexity and memory complexity of the 3/4-round collision attacks are very high, i.e. larger than 2^{120}. From a practical view, it seems that these attacks may be slower than the generic attack. In any case, all the collision attacks are far from being practical, even for 2 rounds.

Table 1: Summary of collision attacks on ASCON-HASH

Attack Type	Rounds	Time complexity	Memory Complexity	Reference
collision attack	2	2^{125*}	negligible	[21]
	2	2^{103}	negligible	[12]
	2	$2^{62.6}$	negligible	Sect. 4
	3	$2^{121.85}$	2^{121}	[19]
	4	$2^{126.77}$	2^{126}	[19]

* The characteristic used is invalid.

Our contributions. We aim to significantly improve the time complexity of the 2-round collision attack in [12] such that it can be much closer to a practical attack. Our contributions are summarized below:

1. We found that the 2-round collision attack in [12] is quite straightforward, i.e., the authors found a better characteristic but did not optimize the attack strategy. Hence, we are motivated to take a closer look at the used 2-round differential characteristic and aim to improve the attack by using some algebraic properties of the S-box as in the recent algebraic attack on LowMC [14, 17], i.e., we are interested in the relations between the difference transitions and value transitions.
2. Based on our findings of the properties of the S-box, we propose to use a better attack framework and advanced algebraic techniques to improve the 2-round collision attack. As a result, the time complexity is reduced from 2^{103} to $2^{62.6}$, as shown in Table 1.

Organization of this paper. In section 2, we define some notations that will be used throughout the paper and briefly describe ASCON-HASH. In section 3, we describe the collision attack framework that will be used in the new attacks. In section 4, we show how to optimize the existing 2-round collision attack with advanced algebraic techniques. Finally, the paper is concluded in section 5.

2 Preliminaries

2.1 Notations

The notations used in this paper are summarized in Table 2.

Table 2: Notations

r	the length of the rate part for ASCON-HASH, $r = 64$
c	the length of the capacity part for ASCON-HASH, $c = 256$
S_j^i	the input state of round i when absorbing the message block M_j
$S^i[j]$	the j-th word (64-bit) of S_i
$S^i[j][k]$	the k-th bit of $S^i[j]$, $k = 0$ means the least significant bit and k is within modulo 64
x_i	the i-th bit of a 5-bit value x, x_0 represents the most significant bit
M	message
M_i	the i-th block of the padded message
$\ggg$	right rotation (circular right shift)
$a\%b$	$a \mod b$
0^n	a string of n zeroes

2.2 Description of ASCON-HASH

The ASCON family offers 2 important hash functions: ASCON-HASH and ASCON-XOF. ASCON-HASH is a sponge-based hash function [2]. In its core, it is a 12-round permutation P^a over a state of 320 bits. The hashing mode is shown in Figure 1.

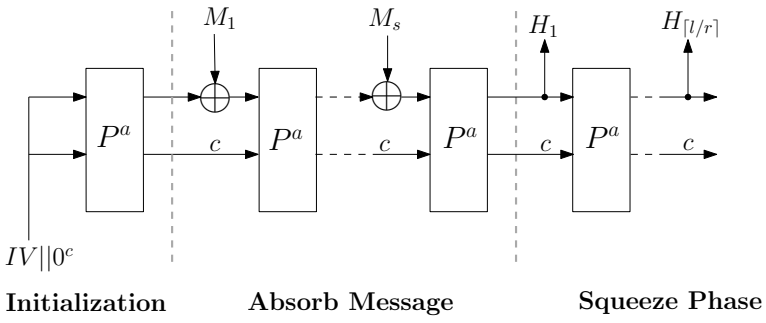

Fig. 1: The mode of ASCON-HASH

For ASCON-HASH, the state denoted by X is divided into five 64-bit words, i.e., $X = X_0||X_1||X_2||X_3||X_4$. The first 64-bit word X_0 will be loaded in the rate part while the remaining 4 words (X_1, X_2, X_3, X_4) are loaded in the capacity part. The round function $f = f_L \circ f_S \circ f_C$ is composed of 3 operations: f_C is

the constant addition, f_S is the substitution layer, and f_L is the linear diffusion layer. For simplicity, the ℓ-round ASCON permutation is simply denoted by f^ℓ.

On the internal states. When absorbing the message block M_j, denote the 320-bit input state at round i ($0 \le i \le 11$) by S_j^i and the state transitions are described below.

$$S_j^i \xrightarrow{f_C} S_j^{i,a} \xrightarrow{f_S} S_j^{i,s} \xrightarrow{f_L} S_j^{i+1}.$$

Note that if we only consider one message block, we simply omit j as below:

$$S^i \xrightarrow{f_C} S^{i,a} \xrightarrow{f_S} S^{i,s} \xrightarrow{f_L} S^{i+1}.$$

The corresponding graphic explanations can be referred to Fig. 2 and Fig. 3, respectively.

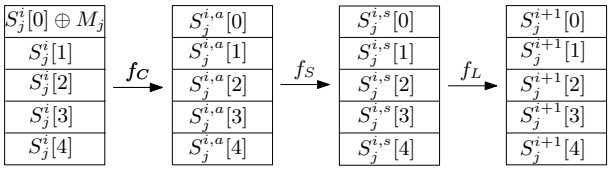

Fig. 2: The 1-round state transition when absorbing M_j

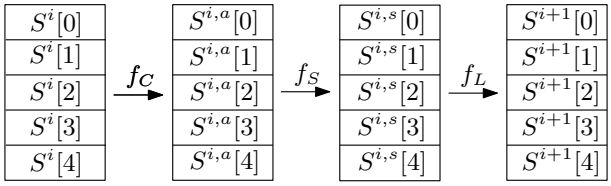

Fig. 3: The 1-round state transition

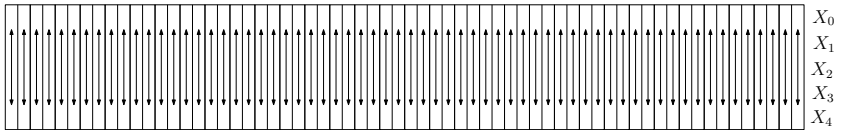

Fig. 4: The substitution layer

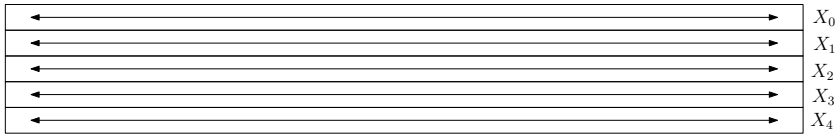

Fig. 5: The linear diffusion layer

Constant addition f_C. For this operation, an 8-bit round constant c_i is added to the word X_2, i.e., $X_2 \leftarrow X_2 \oplus c_i$. The round constants $(c_i)_{0 \le i \le 11}$ for 12-round ASCON-HASH are shown in Table 3.

Table 3: The round constants c_i

i	0	1	2	3	4	5	6	7	8	9	10	11
c_i	0xf0	0xe1	0xd2	0xc3	0xb5	0xa5	0x96	0x87	0x78	0x69	0x5a	0x4b

Substitution layer f_S. At this operation, the state will be updated by 64 parallel applications of a 5-bit S-box. The S-box $(y_0, \ldots, y_4) = \texttt{SB}(x_0, \ldots, x_4)$ is defined as follows:

$$\begin{cases} y_0 = x_4x_1 \oplus x_3 \oplus x_2x_1 \oplus x_2 \oplus x_1x_0 \oplus x_1 \oplus x_0, \\ y_1 = x_4 \oplus x_3x_2 \oplus x_3x_1 \oplus x_3 \oplus x_2x_1 \oplus x_2 \oplus x_1 \oplus x_0, \\ y_2 = x_4x_3 \oplus x_4 \oplus x_2 \oplus x_1 \oplus 1, \\ y_3 = x_4x_0 \oplus x_4 \oplus x_3x_0 \oplus x_3 \oplus x_2 \oplus x_1 \oplus x_0, \\ y_4 = x_4x_1 \oplus x_4 \oplus x_3 \oplus x_1x_0 \oplus x_1. \end{cases} \tag{1}$$

As shown in Fig. 4, the input $(x_0, \ldots, x_4)$ and output $(y_0, \ldots, y_4)$ correspond to one column of the state.

Linear diffusion layer f_L. This operation is used to diffuse each 64-bit word X_i, as shown in Fig. 5. Specifically, X_i is updated by the function $\sum_i$ where $0 \le i \le 4$, as specified below:

$$\begin{cases} X_0 \leftarrow \Sigma_0(X_0) = X_0 \oplus (X_0 \ggg 19) \oplus (X_0 \ggg 28), \\ X_1 \leftarrow \Sigma_1(X_1) = X_1 \oplus (X_1 \ggg 61) \oplus (X_1 \ggg 39), \\ X_2 \leftarrow \Sigma_2(X_2) = X_2 \oplus (X_2 \ggg 1) \oplus (X_2 \ggg 6), \\ X_3 \leftarrow \Sigma_3(X_3) = X_3 \oplus (X_3 \ggg 10) \oplus (X_3 \ggg 17), \\ X_4 \leftarrow \Sigma_4(X_4) = X_4 \oplus (X_4 \ggg 7) \oplus (X_4 \ggg 41). \end{cases}$$

On the initial value and state. The hash function initializes the 320-bit state using a constant $IV =$ `0x00400c0000000000`. Then, the 12-round ASCON permutation is applied and we obtain an initial state $S_1^0 = f^{12}(IV||0^{256})$, as specified below:

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0xee9398aadb67f03d
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$$S_1^0 \leftarrow \begin{matrix} \texttt{0x8bb21831c60f1002} \\ \texttt{0xb48a92db98d5da62} \\ \texttt{0x43189921b8f8e3e8} \\ \texttt{0x348fa5c9d525e140} \end{matrix}$$

The padding rule of ASCON-HASH is as follows: it appends a single 1 and the smallest number of zeroes to M such that the size of padded message in bits is a multiple of $r = 64$. The complete description of the hashing function is given in Algorithm 1 in Appendix A.

3 The Attack Frameworks

For differential-based collision attacks on a sponge-based hash function, one essential step is to find a collision-generating differential characteristic. The second step is to find conforming message pairs satisfying this differential characteristic.

With the development of automatic tools, there are many possible methods to search for a desired differential characteristic. However, when it comes to the second step, i.e., satisfying the conditions of the differential characteristic, it always involves dedicated efforts and sometimes requires nontrivial techniques. For example, the linearization techniques for the KECCAK round function have been widely used to speed up the differential-based collision attack on KECCAK, e.g., the 1/2/3-round connectors [5, 18, 20]. As can be seen from the current record of the Keccak crunchy crypto collision contest[5], it is quite challenging to analyze sponge-based hash functions with a small rate, which is exactly the case of ASCON. It is thus not surprising to see that the best differential-based collision attack on ASCON could only reach up to 2 rounds.

For a sponge-based hash function with a small rate, one main obstacle exists in the available degrees of freedom in each message block. For ASCON, each message block only provides at most 64 free bits. However, for a differential characteristic used for collision attacks, there may exist more than 128 bit conditions, which directly makes it mandatory to utilize at least 3 message blocks.

Let us consider a general case and suppose that we have an ℓ-round collision-generating differential characteristic. Furthermore, suppose we will use k message blocks $(M_1, \ldots, M_k)$ to fulfill the conditions, i.e., we aim to find $(M_1, \ldots, M_k)$ and $(M_1, \ldots, M_{k-1}, M'_k)$ such that

$$S_{j+1}^0 = f^\ell\left(S_j^0 \oplus (M_j||0^{256})\right) \text{ where } 1 \le j \le k-1,$$
$$\star||0^{256} = f^\ell\left(S_k^0 \oplus (M_k||0^{256})\right) \oplus f^\ell\left(S_k^0 \oplus \texttt{SB}(M'_k||0^{256})\right),$$

where $M_k \neq M'_k$ and $\star$ is an arbitrary r-bit value.

[5] `https://keccak.team/crunchy_contest.html`

From the differential characteristic, suppose that there are n_c bit conditions on the capacity part of S_k^0 and the remaining conditions hold with probability 2^{-n_k}. Then, a straightforward method to find conforming message pairs is as follows:

Step 1: Find a solution of $(M_1, \ldots, M_{k-1})$ such that the n_c bit conditions on the capacity part of S_k^0 can hold.
Step 2: Exhaust M_k and check whether remaining n_k bit conditions can hold. If there is a solution, a collision is found. Otherwise, return to Step 1.

For convenience, we call the above procedure *the general 2-step attack framework*. Note that this has been widely used and it is really not a new idea.

For a sponge with rate r, we need to perform Step 2 for about 2^{n_k-r} times and hence we need to perform Step 1 for 2^{n_k-r} times. Suppose the time complexity to find a solution of $(M_1, \ldots, M_{k-1})$ and M_k is $T_{\mathtt{pre}}$ and $T_{\mathtt{k}}$, respectively. In this way, the total time complexity $T_{\mathtt{total}}$ is estimated as

$$T_{\mathtt{total}} = (k-1) \cdot 2^{n_k-r} \cdot T_{\mathtt{pre}} + 2^{n_k-r} \cdot T_{\mathtt{k}}. \tag{2}$$

If $T_{\mathtt{k}}$ and $T_{\mathtt{pre}}$ are simply treated as 2^r and 2^{n_c}, respectively, i.e., only the naive exhaustive search is performed, then

$$T_{\mathtt{total}} = (k-1) \cdot 2^{n_k+n_c-r} + 2^{n_k}.$$

In other words, the total time complexity is directly related to the probability of the differential characteristic, i.e., $2^{-n_c-n_k}$.

In many cases, the attackers can optimize $T_{\mathtt{k}}$ by using some advanced techniques to satisfy partial conditions implied in the differential characteristic, i.e., $T_{\mathtt{k}}$ can be smaller than 2^r. For example, the target difference algorithm proposed in [5] is one of such techniques. However, to optimize $T_{\mathtt{pre}}$, one has to solve a problem similar to the $\ell-$round preimage finding problem. In most cases, this is not optimized due to the increasing difficulty and it is simply treated as $T_{\mathtt{pre}} = 2^{n_c}$.

3.1 The Literature and Our New Strategy

It is found that neither $T_{\mathtt{k}}$ nor $T_{\mathtt{pre}}$ has been optimized for the existing 2-round collision attacks on ASCON-HASH [12, 21] and they exactly follow the above attack framework. In the collision attack on 6-round GIMLI-HASH [13], the attackers optimized both $T_{\mathtt{k}}$ and $T_{\mathtt{pre}}$ where $k = 2$.

As can be noted in our new attacks on ASCON-HASH, optimizing $T_{\mathtt{k}}$ is indeed quite straightforward after a little deeper analysis of the round function and its 5-bit S-box. However, optimizing $T_{\mathtt{pre}}$ looks infeasible at the first glance. Indeed even if $T_{\mathtt{k}}$ is optimized to 1, the improved factor is still quite small. Therefore, to achieve significant improvements, it is necessary to optimize $T_{\mathtt{pre}}$.

Our idea to achieve this purpose is to further convert the n_c conditions on the capacity part of S_k^0 into some n_c^1 conditions on the capacity part of S_{k-1}^0, as Fig. 6 shows. In this way, our attack is stated as follows:

Step 1: Find a solution of $(M_1, \ldots, M_{k-2})$ such that the n_c^1 bit conditions on the capacity part of S_{k-1}^0 can hold.
Step 2: Enumerate all the solutions of M_{k-1} such that the conditions on the capacity part of S_k^0 can hold.
Step 3: Exhaust M_k and check whether remaining n_k bit conditions can hold. If there is a solution, a collision is found. Otherwise, return to Step 1.

To distinguish this from *the general 2-step attack framework*, we call the above procedure *the general 3-step attack framework*.

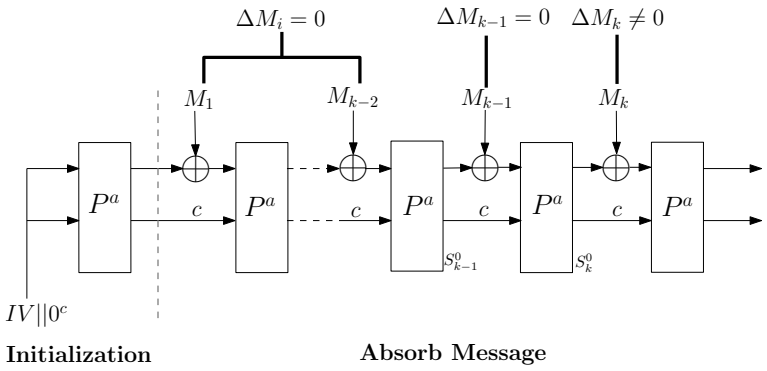

Fig. 6: The general 3-step attack framework

Analysis of the time complexity. For convenience, the time complexity of Step 1, 2 and 3 is denoted by $T_{\texttt{pre1}}$, $T_{\texttt{k-1}}$ and $T_{\texttt{k}}$, respectively. In this way, the total time complexity becomes

$$T_{\texttt{total}} = (k-2) \cdot 2^{n_k+n_c-2r} \cdot T_{\texttt{pre1}} + 2^{n_k+n_c-2r} \cdot T_{\texttt{k-1}} + 2^{n_k-r} \cdot T_{\texttt{k}}. \quad (3)$$

Specifically, we need on average 2^{n_k-r} different valid solutions of $(M_1, \ldots, M_{k-1})$. In this sense, we need about $2^{n_k+n_c-2r}$ different valid solutions of $(M_1, \ldots, M_{k-2})$ because for each valid $(M_1, \ldots, M_{k-2})$, we expect to have 2^{r-n_c} valid solutions of M_{k-1}.

Based on Equation 3, if $n_c < r$ holds, we have

$$2^{n_k+n_c-2r} < 2^{n_k-r}.$$

Compared with Equation 2, this case has indicated the possibility to optimize the attack if $T_{\texttt{k-1}}$ can be significantly optimized and $T_{\texttt{pre1}}$ is relatively small, i.e., we know $T_{\texttt{pre1}} \leq 2^{n_c'}$.

On the purpose to convert conditions. As stated above, we have to optimize $T_{\texttt{pre1}}$. This is related to the original purpose to introduce conditions on the capacity

part of S^0_{k-1}. Specifically, we expect that after adding these conditions, we can efficiently enumerate the solutions of M_{k-1} to satisfy the n_c conditions on the capacity part of S^0_k. In other words, without these conditions, we still can only perform the naive exhaustive search over M_{k-1} and no improvement can be obtained, i.e., the time complexity is

$$\begin{aligned}&(k-2)\cdot 2^{n_k+n_c-2r}+2^{n_k+n_c-2r}\cdot 2^r+2^{n_k-r}\cdot T_{\mathtt{k}}\\=\;&(k-2)\cdot 2^{n_k+n_c-2r}+2^{n_k+n_c-r}+2^{n_k-r}\cdot T_{\mathtt{k}}.\end{aligned}$$

The big picture of our new attacks. In our attacks, we do not make more efforts to convert the n'_c conditions on S^0_{k-1} into conditions on the previous input states due to the increasing difficulty. Hence, in our setting, we will make

$$T_{\mathtt{pre1}}=2^{n'_c}.$$

In this way, the total time complexity is estimated as

$$T_{\mathtt{total}}=(k-2)\cdot 2^{n_k+n_c+n'_c-2r}+2^{n_k+n_c-2r}\cdot T_{\mathtt{k\text{-}1}}+2^{n_k-r}\cdot T_{\mathtt{k}}. \tag{4}$$

In the following, we will describe how to significantly optimize $T_{\mathtt{k\text{-}1}}$ and $T_{\mathtt{k}}$ based on an existing 2-round differential characteristic of ASCON.

4 Collision Attacks on 2-Round ASCON-HASH

The collision attack in this paper is based on the 2-round differential characteristic proposed in [12], as shown in Table 4. Note that the first collision attack on 2-round ASCON-HASH was proposed in [21] but the differential characteristic is shown to be invalid in [13]. We have verified with the technique in [13] that the 2-round differential characteristic in [12] is correct.

Table 4: The 2-round differential characteristic in [12]

ΔS^0 (2^{-54})	ΔS^1 (2^{-102})	ΔS^2
0xbb450325d90b1581	0x2201080000011080	0xbaf571d85e1153d7
0x0	0x2adf0c201225338a	0x0
0x0	0x0	0x0
0x0	0x0000000100408000	0x0
0x0	0x2adf0c211265b38a	0x0

According to [12], there are 27 and 28 active S-boxes in the first and second round, respectively. Specifically, there are 54 bit conditions on the capacity part of the input S^0 and 102 bit conditions on the input state S^1 of the second round. With our notations, there are

$$n_c=54,\quad n_k=102.$$

With this differential characteristic, they used the technique in [21] to mount the collision attack with $k = 4$ message blocks and its time complexity is 2^{102}. It follows the general 2-step attack framework described above without optimization on $T_{\mathtt{pre}}$ and $T_{\mathtt{k}}$, i.e.,

$$T_{\mathtt{pre}} = 2^{54}, \quad T_{\mathtt{k}} = 2^{64}.$$

In this way, the total time complexity can be computed based on Equation 2, i.e.,

$$T_{\mathtt{total}} = 2 \times 2^{102-64} \times 2^{54} + 2^{102-64} \times 2^{64} = 2^{93} + 2^{102} \approx 2^{102}. \tag{5}$$

It should be noted that in [12], the authors simply checked whether M_3 and $M_3 \oplus \Delta S_3^0$ can follow the 2-round differential characteristic by exhausting M_3 and hence the time complexity in [12] is estimated as $2 \times 2^{102} = 2^{103}$. In other words, they do not take the specific conditions into account, while in the above, we only check whether the conditions on the S^1 hold for each M_3.

4.1 Optimizing $T_{\mathtt{k}}$ Using Simple Linear Algebra

Indeed, it is quite straightforward to optimize $T_{\mathtt{k}}$. However, even if it is reduced to 1, the time complexity is still high, i.e., 2^{92} according to Equation 5. Let us elaborate on how to significantly optimize $T_{\mathtt{k}}$ in this section. First, we need to study some properties of the S-box.

Studying the active S-boxes in the first round. First, we describe why there are 54 bit conditions on the capacity part of S^0.

Property 1 *[21] For an input difference $(\Delta_0, \ldots, \Delta_4)$ satisfying $\Delta x_1 = \Delta x_2 = \Delta x_3 = \Delta x_4 = 0$ and $\Delta x_0 = 1$, the following constraints hold:*

- *For the output difference:*

$$\begin{cases} \Delta y_0 \oplus \Delta y_4 = 1, \\ \Delta y_1 = \Delta x_0, \\ \Delta y_2 = 0. \end{cases} \tag{6}$$

- *For the input value:*

$$\begin{cases} x_1 = \Delta y_0 \oplus 1, \\ x_3 \oplus x_4 = \Delta y_3 \oplus 1. \end{cases} \tag{7}$$

Based on Property 1 and the 2-round differential characteristic in Table 4, we can derive $27 + 27 = 54$ bit conditions on the capacity part of S^0, i.e., 27 bit conditions on $S^0[1]$ and 27 bit conditions on $S^0[3] \oplus S^0[4]$. This also explains why $n_c = 54$.

Studying the active S-boxes in the second round. As the next step, we further study the 28 active S-boxes in the second round. We observe that from ΔS^1 to $\Delta S^{1,s}$, there are only 3 different possible difference transitions $(\Delta x_0, \ldots, \Delta x_4) \rightarrow (\Delta y_0, \ldots, \Delta y_4)$ through the S-box, as shown below:

$$\begin{aligned}(1,1,0,0,1) &\rightarrow (1,0,0,0,0),\\ (0,0,0,1,1) &\rightarrow (1,0,0,0,0),\\ (0,1,0,0,1) &\rightarrow (1,0,0,0,0).\end{aligned}$$

Similar to the algebraic attacks on LowMC [14, 17], we study and exploit the properties of the $(x_0, \ldots, x_4)$ such that

$$\mathtt{SB}(x_0, \ldots, x_4) \oplus \mathtt{SB}(x_0 \oplus \Delta x_0, \ldots, x_4 \oplus \Delta x_4) = (\Delta y_0, \ldots, \Delta y_4) = (1,0,0,0,0)$$

where

$$(\Delta x_0, \ldots, \Delta x_4) \in \{(1,1,0,0,1), (0,0,0,1,1), (0,1,0,0,1)\}.$$

It is found that

- for $(\Delta x_0, \ldots, \Delta x_4) = (1,1,0,0,1)$, all possible $(x_0, \ldots, x_4)$ form an affine subspace of dimension 2, as shown below:

$$x_0 \oplus x_4 = 0, \quad x_1 = 1, \quad x_3 = 0; \tag{8}$$

- for $(\Delta x_0, \ldots, \Delta x_4) = (0,0,0,1,1)$, all possible $(x_0, \ldots, x_4)$ form an affine subspace of dimension 2, as shown below:

$$x_1 = 0, \quad x_2 = 0, \quad x_3 \oplus x_4 = 0; \tag{9}$$

- for $(\Delta x_0, \ldots, \Delta x_4) = (0,1,0,0,1)$, all possible $(x_0, \ldots, x_4)$ form an affine subspace of dimension 1, as shown below:

$$x_0 = 0, \quad x_1 \oplus x_4 = 1, \quad x_2 = 0, \quad x_3 = 0. \tag{10}$$

As a result, the difference transitions in the second round, i.e., the 28 active S-boxes, directly impose 102 *linear conditions* on S^1. Note that it is unclear whether the probability 2^{-102} is directly computed according to the differential distribution table (DDT) of the 5-bit S-box in [12]. At least, we do not see any such related claims in [12] that the probability 2^{-102} is caused by 102 linear conditions on S^1, i.e., the conditions may be *nonlinear* if we do not carefully study the relations between the difference transitions and values transitions. Indeed, we can simply generalize the above observations for any degree-2 S-box, as shown in Appendix B, i.e. all the conditions on the input bits must be linear for each valid difference transition of a degree-2 S-box.

More nonlinear conditions on the capacity part of S^0. As can be noted from Equation 9 and Equation 10, there will be conditions on $S^1[2]$, i.e., the conditions on x_2 in Equation 9 and Equation 10. However, according to the definition of the S-box, we know that

$$y_2 = x_4x_3 \oplus x_4 \oplus x_2 \oplus x_1 \oplus 1.$$

Hence, after the capacity part of S_3^0 is fixed, $S^1[2]$ is irrelevant to $S^0[0]$. As a result, apart from the 54 linear conditions on the capacity part of S^0, there are also 21 nonlinear (quadratic) conditions on the capacity part of S^0. In other words, at the first glance, although there are 102 linear conditions on S^1, there are indeed only $102 - 21 = 81$ linear conditions on S^1 depending on $S^0[0]$ after the capacity part of S^0 is known. Hence, we can equivalently say that

$$n_c = 54 + 21 = 75, \quad n_k = 81.$$

With the general 2-step attack framework, the total time complexity is not affected as $n_c + n_k$ remains the same, i.e., it is still 2^{102}.

Optimizing $T_{\mathtt{k}}$. After knowing that there are 81 linear conditions on S^1 depending on $S^0[0]$ after the capacity part of S^0 is known, optimizing $T_{\mathtt{k}}$ is quite straightforward. Recall the general 2-step attack framework described previously. Specifically, by using 3 message blocks (M_1, M_2, M_3), we first generate valid (M_1, M_2) such that the 75 bit conditions on the capacity part of S_3^0 can hold. Then, since M_3 is only added to $S_3^0[0]$, S_3^1 directly becomes linear in M_3 and we know there are 81 linear conditions on S_3^1. Therefore, we can construct 81 linear equations in M_3, i.e., 64 variables. Similar to the idea in [15], solving this linear equation system is equivalent to exhausting all possible values of M_3 and hence $T_{\mathtt{k}}$ is reduced to the time complexity to solve 81 linear equations in 64 variables that requires $81 \times 81 \times 64 \approx 2^{19}$ bit operations. As explained before, only optimizing $T_{\mathtt{k}}$ is insufficient to significantly improve the attack and we need to further optimize $T_{\mathtt{pre}}$.

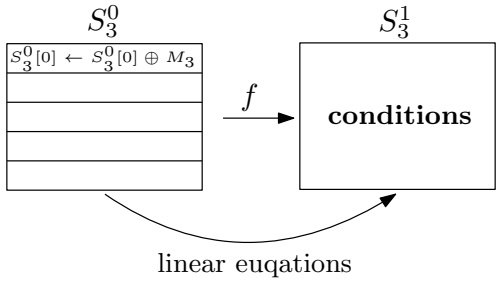

Fig. 7: Exhaust M_3 by solving linear equations

4.2 Finding Valid (M_1, M_2) with Advanced Techniques

To find valid (M_1, M_2), we are now only simply looping over (M_1, M_2) and checking whether the 75 bit conditions on the capacity part can hold. To improve the attack, we have to avoid such a naive loop. In what follows, we describe how to use the general 3-step attack framework stated above to overcome this obstacle.

The core idea is to utilize a 2-round property of ASCON. Let us explain it step by step.

Property 2 *For* $(y_0, \ldots, y_4) = \texttt{SB}(x_0, \ldots, x_4)$*, if* $x_3 \oplus x_4 = 1$*,* y_3 *will be independent to* x_0*.*

Proof. We can rewrite y_3 as follows:

$$y_3 = (x_4 \oplus x_3 \oplus 1)x_0 \oplus (x_4 \oplus x_3 \oplus x_2 \oplus x_1).$$

Hence, if $x_3 \oplus x_4 = 1$, y_3 is irrelevant to x_0.

Property 3 *Let*

$$(S^1[0], \ldots, S^1[4]) = f(S^0[0], \ldots, S^0[4]), \quad (S^2[0], \ldots, S^2[4]) = f(S^1[0], \ldots, S^1[4]),$$

where $(S^0[1], S^0[2], S^0[3], S^0[4])$ *are constants and* $S^0[0]$ *is the only variable. Then, it is always possible to make* u *bits of* $S^2[1]$ *linear in* $S^0[0]$ *by adding at most* $9u$ *bit conditions on* $S^0[3] \oplus S^0[4]$*.*

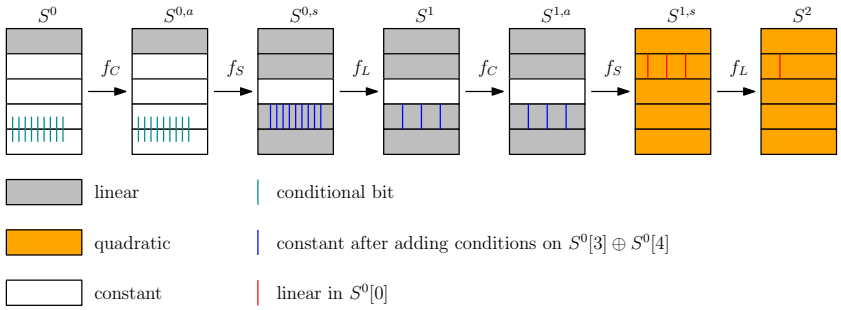

Fig. 8: Adding conditions on the capacity part to linearize $S^2[1]$

Proof. First, since $S^0[0]$ is the only variable, according to the definition of f, we know that $(S^1[0], S^1[1], S^1[3], S^1[4])$ are linear in $S^0[0]$ while $S^1[2]$ is still constant.

Each bit $S^2[1][i]$ can be expressed as

$$S^2[1][i] = S^{1,s}[1][i] \oplus S^{1,s}[1][i+61] \oplus S^{1,s}[1][i+39].$$

To make $S^2[1][i]$ linear in $S^0[0]$, we need to ensure

$$S^{1,s}[1][i] \oplus S^{1,s}[1][i+61] \oplus S^{1,s}[1][i+39]$$

is linear in $S^0[0]$. According to the definition of the S-box specified in Equation 1, the expression of y_1 is

$$y_1 = x_4 \oplus x_1x_3 \oplus x_3 \oplus x_2(x_3 \oplus x_1 \oplus 1) \oplus x_1 \oplus x_0.$$

Hence, if x_2 is constant, there is only one quadratic term x_1x_3 in the expression of y_1.

According to the above analysis, $S^1[2]$ is always constant. Hence, we have

$$\begin{aligned} & S^{1,s}[1][i] \oplus S^{1,s}[1][i+61] \oplus S^{1,s}[1][i+39] \\ = & S^1[1][i]S^1[3][i] \oplus S^1[1][i+61]S^1[3][i+61] \oplus S^1[1][i+39]S^1[3][i+39] \\ \oplus & L_i(S^1[0], \ldots, S^1[4]) \end{aligned} \tag{11}$$

where L_i is a linear function.

Furthermore, according to Property 2, we can make $S^{0,s}[3][i]$ $(0 \le i \le 63)$ irrelevant to $S^0[0]$ by adding 1 bit condition on $S^0[3] \oplus S^0[4]$. In this way, we can add at most 9 bit conditions on $S^0[3] \oplus S^0[4]$ to make $(S^1[3][i], S^1[3][i+61], S^1[3][i+39])$ irrelevant to $S^0[0]$ since each bit of $S^1[3]$ is linear in 3 bits of $S^{0,s}[3]$. Once $(S^1[3][i], S^1[3][i+61], S^1[3][i+39])$ is irrelevant to $S^0[0]$, $S^{1,s}[1][i] \oplus S^{1,s}[1][i+61] \oplus S^{1,s}[1][i+39]$ becomes linear in $S^0[0]$ according to Equation 11. Hence, to make u bits of $S^2[1]$ linear in $S^0[0]$, we need to add at most $9u$ bit conditions on $S^0[3] \oplus S^0[4]$.

A graphical explanation for Property 3 can be seen from Fig. 8.

Property 4 *Let*

$$(S^1[0], \ldots, S^1[4]) = f(S^0[0], \ldots, S^0[4]), \quad (S^2[0], \ldots, S^2[4]) = f(S^1[0], \ldots, S^1[4]),$$

where $(S^0[1], S^0[2], S^0[3], S^0[4])$ *are constants and* $S^0[0]$ *is the only variable. Then, it is always possible to make* u *bits of* $S^2[1]$ *linear in* $S^0[0]$ *by guessing* $3u$ *linear equations in* $S^0[0]$.

Proof. Similar to the proof of Property 3, we have

$$\begin{aligned} & S^{1,s}[1][i] \oplus S^{1,s}[1][i+61] \oplus S^{1,s}[1][i+39] \\ = & S^1[1][i]S^1[3][i] \oplus S^1[1][i+61]S^1[3][i+61] \oplus S^1[1][i+39]S^1[3][i+39] \\ \oplus & L_i(S^1[0], \ldots, S^1[4]) \end{aligned}$$

where L_i is a linear function and $(S^1[0], S^1[1], S^1[2], S^1[3], S^1[4])$ are linear in $S^0[0]$. Hence, if we guess $(S^1[3][i], S^1[3][i+61], S^1[3][i+39])$, $S^2[1][i]$ will be linear in $S^0[0]$. In other words, by guessing 3 linear equations in $S^0[0]$, $S^2[1][i]$ can be linear in $S^0[0]$.

A graphical explanation for Property 4 can be seen from **??**.

Improving the attack. Based on the above discussions, it is now possible to further improve the 2-round collision attack. We utilize the general 3-step attack framework where $k = 3$, i.e., we use message blocks (M_1, M_2, M_3). From previous analysis, there are 54 linear conditions on the capacity part of S_3^0 and among them, 27 bit conditions are on $S_3^0[1]$ (or $S_2^2[1]$). Based on Property 3 and Property 4, it is possible to satisfy these 54 linear conditions more efficiently with advanced algebraic techniques, i.e., we can improve $T_{\texttt{k-1}}$. We emphasize that there are additional 21 quadratic conditions on the capacity part of S_3^0, but we will not consider them to speed up the exhaustive search over M_2 due to the increasing difficulty, i.e., it is required to solve degree-4 Boolean equations.

Specifically, based on Property 3, we can add $9u_1$ conditions on the capacity part of S_2^0 such that u_1 bits of $S_3^0[1]$ can be linear in M_2 after the capacity part of S_2^0 is known. Moreover, based on Property 4, after the capacity part of S_2^0 is known, we can guess $3u_2$ linear equations in M_2 such that u_2 bits of $S_3^0[1]$ can be linear in M_2. In total, we set up u_1+4u_2 linear equations in 64 variables to satisfy $u_1 + u_2$ out of 27 bit conditions. Then, we perform the Gaussian elimination on these $u_1 + 4u_2$ linear equations and obtain

$$u_3 = 64 - u_1 - 4u_2$$

free variables.

Note that the first round is always freely linearized and the remaining $54 - u_1 - u_2$ linear conditions on S_3^0 can be expressed as quadratic equations in these u_3 free variables. In a word, to efficiently exhaust M_2 such that the 54 conditions on S_3^0 can hold, we can perform the following procedure:

Step 1: Guess $3u_2 = 42$ bits of M_2 and construct $4u_2 + u_1$ linear equations.
Step 2: Apply the Gaussian elimination to the system and obtain $u_3 = 64 - u_1 - 4u_2$ free variables.
Step 3: Construct $54 - u_1 - u_2$ quadratic equations in these u_3 variables and solve the equations.
Step 4: Check whether the remaining 21 quadratic conditions on the capacity part of S_3^0 can hold for each obtained solution.

We use a similar method in [3, 16] to estimate the time complexity to solve a quadratic equation system. After some calculations, the optimal choice of (u_1, u_2, u_3) is as follows:

$$u_1 = 3, \quad u_2 = 13 \quad u_3 = 9.$$

In other words, we need to perform the Gaussian elimination on 55 linear equations in 64 variables for $2^{3u_2} = 2^{39}$ times. Then, we need to solve 38 quadratic equations in 9 variables for 2^{39} times. The total time complexity is estimated as

$$2^{39} \times (55^2 \times 64 + 38^2 \times 45) \approx 2^{56.6}$$

bit operations. The cost of Step 4 is negligible since it is expected to perform such a check for about $2^{64-54} = 2^{10}$ times.

Time complexity evaluation. Based on the previous general 3-step attack framework using 3 message blocks (M_1, M_2, M_3), we have $9u_1 = 27$ conditions on S_2^0 and we need $2^{81+75-128} = 2^{28}$ different valid M_1. The cost of this step can be estimated as $2^{28+27} = 2^{55}$ calls to the 2-round ASCON permutation. Then, for each valid M_1, i.e., each valid S_2^0, we can exhaust M_1 with $2^{56.6}$ bit operations. At last, for each valid (M_1, M_2), we can exhaust M_3 with about 2^{19} bit operations. Assume that one round of the ASCON permutation takes about $15 \times 64 \approx 2^{10}$ bit operations, the total time complexity can be estimated as

$$T_{\texttt{total}} = 2^{28} \times 2^{27} + 2^{28} \times 2^{56.6-11} + 2^{17} \times 2^{19-11} \approx 2^{73.6}$$

calls to the 2-round ASCON permutation.

4.3 Further Optimizing the Guessing Strategy

In the above improved 2-round collision attack, we mainly exploit Property 3 and Property 4 to make some conditional bits of $S_2^2[1]$ linear in M_2. Specifically, the core problem is to make

$$(S_2^1[3][i], S_2^1[3][i+61], S_2^1[3][i+39])$$

constant by either guessing their values according to Property 4 or adding conditions on $S_2^0[3] \oplus S_2^0[4]$ according to Property 3. However, the two strategies are independently used for different bits of $S_2^2[1]$. It can be noted that for one specific conditional bit of $S_2^2[1]$, i.e., $S_2^2[1][i]$, we can guess g out of 3 bits of $(S_2^1[3][i], S_2^1[3][i+61], S_2^1[3][i+39])$ and add $3 \times (3-g)$ conditions on $S_2^0[3] \oplus S_2^0[4]$ to achieve the same goal. In other words, for the same conditional bit, we can use a hybrid guessing strategy.

As the next step, we aim to optimize the guessing strategy such that we can obtain a sufficient number of linear equations by guessing a smaller number of linear equations or adding a smaller number of extra conditions on $S_2^0[3] \oplus S_2^0[4]$. For example, for the above naive guess strategy, we need to add $9u_1 = 27$ bit conditions on $S_2^0[3] \oplus S_2^0[4]$ and we need to further guess $3u_2 = 39$ linear equations in order to get $u_1 + 4u_2 = 3 + 52 = 55$ linear equations in M_2. Can we guess fewer bits to achieve better results?

Note that there are 27 conditional bits in $S_2^2[1]$. For completeness, we denote the set of i such that $S_2^2[1][i]$ is conditional by $\mathcal{I}$ and we have

$$\begin{aligned}\mathcal{I} = \{&0, 7, 8, 10, 12, 16, 17, 19, 24, 27, 28, 30, 31, 32, 34, 37,\\ &40, 41, 48, 50, 54, 56, 57, 59, 60, 61, 63\}.\end{aligned}$$

For each $i \in \mathcal{I}$, let

$$\mathcal{P}_i = \{i, (i+61)\%64, (i+39)\%64\}.$$

Further, let

$$\mathcal{P}_i = \mathcal{P}_{i,g} \cup \mathcal{P}_{i,a}, \quad \mathcal{P}_{i,g} \cap \mathcal{P}_{i,a} = \emptyset.$$

In other words, to linearize $S_2^2[1][i]$, we guess $S_2^1[3][j_0]$ where $j_0 \in \mathcal{P}_{i,g}$ and make $S_2^1[3][j_1]$ constant where $j_1 \in \mathcal{P}_{i,a}$ by adding 3 conditions on

$$S_2^0[3][j_1] \oplus S_2^0[4][j_1], \quad S_2^0[3][j_1+10] \oplus S_2^0[4][(j_1+10)], \quad S_2^0[3][j_1+17] \oplus S_2^0[4][(j_1+17)],$$

We can build a simple MILP model to determine the optimal choice of a subset $\mathcal{I}' \subseteq \mathcal{I}$ and the corresponding $\mathcal{P}_{i,g}$ and $\mathcal{P}_{i,a}$ where $i \in \mathcal{I}'$ such that the total time complexity of the attack is optimal. Specifically, assuming that after adding u_4 conditions on $S_2^0[3] \oplus S_2^0[4]$ and guessing u_5 bits of $S_2^1[3]$, we can set up u_6 linear equations for u_6 conditional bits of $S_2^2[1]$. In this way, we have in total $u_5 + u_6$ linear equations and after the Gaussian elimination, we can set up $54 - u_6$ quadratic equations in $u_7 = 64 - u_5 - u_6$ free variables. After some configurations, we propose to choose

$$u_4 = 31, \quad u_5 = 28, \quad u_6 = 27$$

as the optimal parameters. In other words, we can make all the 27 conditional bits of $S_2^2[1]$ linear in M_2 by guessing 28 linear equations in $S_2^1[3]$ and adding 31 bit conditions on $S_2^0[3] \oplus S_2^0[4]$. In this way, we need to perform the Gaussian elimination to $u_5 + u_6 = 55$ linear equations in 64 variables that requires about $2^{17.6}$ bit operations and then solve 27 quadratic equations in $u_7 = 64 - 55 = 9$ variables. Based on the method [3,16] to estimate the time complexity to solve such an overdefined quadratic equation system, it takes about $27^2 \times 45 + 2^3 \times 12^2 \times 6 \approx 2^{15.3}$ bit operations. Hence, the new total time complexity is

$$T_{\texttt{total}} = 2^{28} \times 2^{31} + 2^{28} \times 2^{28} \times (2^{17.6} + 2^{15.3}) \times 2^{-11} + 2^{17} \times 2^{19-11} \approx 2^{62.6}.$$

In conclusion, with the optimal guess strategy and advanced algebraic techniques, we can improve the best collision attack on 2-round ASCON-HASH by a factor of about $2^{40.4}$. For completeness, the required 28 guessed bits of $S_2^1[3]$ and the 31 condition bits of $S_2^0[3] \oplus S_2^0[4]$ are shown in Table 5.

Table 5: The optimal guessing strategy

$\bigcup_{i \in \mathcal{I}} \mathcal{P}_{i,g}$
$\{0, 3, 4, 7, 8, 10, 14, 15, 17, 21, 24, 25, 27, 28, 31, 32, 34, 35, 37, 38, 41, 45, 48, 51, 54, 55, 58, 61\}$
$\bigcup_{i \in \mathcal{I}} \mathcal{P}_{i,a}$
$\{2, 5, 6, 9, 12, 13, 16, 19, 23, 29, 30, 36, 39, 40, 46, 47, 49, 50, 53, 56, 57, 59, 60, 63\}$
$\{j, (j+10)\%64, (j+17)\%64 \mid j \in \bigcup_{i \in \mathcal{I}} \mathcal{P}_{i,a}\}$
$\{0, 2, 3, 5, 6, 9, 10, 12, 13, 15, 16, 19, 22, 23, 26, 29, 30, 33, 36, 39, 40, 46, 47, 49, 50, 53, 56, 57, 59, 60, 63\}$

5 Conclusion

By carefully studying the relations between the difference transitions and values transitions through the S-box, we show that the existing collision attacks on

2-round ASCON-HASH can be significantly improved with the aid of advanced algebraic techniques. We expect our close look at the algebraic properties of the S-box can inspire more efficient attacks on ASCON-HASH or ASCON-XOF.

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A The Algorithmic Description of ASCON-HASH

The Algorithmic Description of ASCON-HASH is shown in Algorithm 1.

Algorithm 1: ASCON-HASH

Input: $M \in \{0,1\}^*$
Output: hash $H \in \{0,1\}^{256}$
Initialization:
$S_1^0 \leftarrow f^{12}(IV||0^c)$;

Absorbing:
$M_1, \ldots, M_s \leftarrow M||1||0^*$;
for $i = 1, \ldots, s$ **do**
  $S_{i+1}^0 \leftarrow f^{12}\left(S_i^0 \oplus (M_i||0^c)\right)$;
end

Squeezing:
$S^0 \leftarrow S_{s+1}^0$;
for $i = 1, \ldots, t = \lceil 256/r \rceil$ **do**
  $H_i \leftarrow S^0[0]$;
  $S^0 \leftarrow f^{12}(S^0)$;
end
return $\lfloor H_1||\ldots||H_t \rfloor_{256}$;

B On Degree-2 S-box

For an n-bit S-box whose algebraic degree is 2, we can show that for any valid pair of input and output difference, the inputs satisfying this difference transition must form an affine subspace.

Let $(x_0, \ldots, x_{n-1}) \in \mathbb{F}_2^n$ and $(y_0, \ldots, y_{n-1}) \in \mathbb{F}_2^n$ be the input and output of the S-box. Further, let

$$y_i = f_i(x_0, \ldots, x_{n-1}),\ 0 \le i \le n-1,$$

where the algebraic degree of f_i is at most 2.

Given any valid input difference $(\Delta x_0, \ldots, \Delta x_{n-1})$ and output difference $(\Delta y_0, \ldots, \Delta y_{n-1})$, we aim to show that $(x_0, \ldots, x_{n-1})$ satisfying the following n equations must form an affine subspace:

$$\begin{aligned} f_0(x_0, \ldots, x_{n-1}) \oplus f_0(x_0 \oplus \Delta x_0, \ldots, x_{n-1} \oplus \Delta x_{n-1}) &= \Delta y_0, \\ &\cdots \\ f_{n-1}(x_0, \ldots, x_{n-1}) \oplus f_{n-1}(x_0 \oplus \Delta x_0, \ldots, x_{n-1} \oplus \Delta x_{n-1}) &= \Delta y_{n-1}. \end{aligned}$$

First, since $(\Delta x_0, \ldots, \Delta x_{n-1}) \to (\Delta y_0, \ldots, \Delta y_{n-1})$ is a valid difference transition, there must exist solutions to the above n equations. We only need to show that all the n equations are indeed linear in $(x_0, \ldots, x_{n-1})$ for each given $(\Delta x_0, \ldots, \Delta x_{n-1}, \Delta y_0, \ldots, \Delta y_{n-1})$ and then the proof is over. Note that the algebraic degree of f_i is at most 2. In this case,

$$f_i(x_0, \ldots, x_{n-1}) \oplus f_i(x_0 \oplus \Delta x_0, \ldots, x_{n-1} \oplus \Delta x_{n-1})$$

must be linear in $(x_0, \ldots, x_{n-1})$, thus completing the proof.